Thermal Dynamics of a Distributed Parameter Nonadiabatic Humidification Process

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Results of a theoretical and experimental study of the thermal dynamics of a distributed parameter nonadiabatic humidification process are presented. A wetted-wall column was selected for study to ensure standard and reproducible velocity profiles and a definable area for heat and mass transfer. Four mathematical models which consider various heat and mass transfer effects were derived in the time domain and solved in the frequency domain. All models, including a simple model previously tested for the adiabatic humidification case, were in good agreement with experimental results obtained by pulse testing.

Few dynamic studies of distributed parameter simultaneous heat and mass transfer systems have been reported. Bruley and Prados (*2*, *3*) performed a frequency response analysis of a wetted-wall adiabatic humidifier. Both laminar and turbulent air flow were studied with water used in countercurrent laminar flow. Lewis (*11*) experimentally studied adiabatic column dynamics by the pulse testing method. Hunt (*10*) extended the frequency range of previous studies and investigated the effect of various input pulse characteristics.

For the adiabatic humidification case, the liquid temperature can be assumed constant and independent of axial position. This means that the mathematical analysis can be restricted to the gas phase alone with a boundary condition of constant temperature at the gas-liquid interface. The present study deals with the thermal dynamics of the nonadiabatic humidification case. Here, nonadiabatic implies the existence of axial temperature gradients in both the gas and liquid phases. Mass transfer and liquid phase dynamics are considered in the theoretical development in addition to the gas phase dynamics.

A wetted wall-column was selected for study because of its comparatively well-defined area for heat and mass transfer. A reasonably accurate estimate of this variable was deemed necessary for testing the theoretical models. An air-water system with turbulent gas and liquid flow was investigated.

THEORETICAL DEVELOPMENT

All mathematical models are based upon the following assumptions:

1. Flat velocity and temperature profiles exist in both phases.
2. Gas phase mass velocity is constant.
3. Film theory applies.
4. Mass transfer sensible heat effect is negligible.
5. Heat loss from the liquid phase to the surroundings is negligible.

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A mathematical description of the system is obtained from energy balances on the gas and liquid phases and the interface. The following dynamic equations for the gas and liquid phase temperature changes are obtained from these energy balances after a sequence of algebraic manipulations:

$$\frac{\partial t_g}{\partial \theta} + V_g \frac{\partial t_g}{\partial z} = \frac{UP}{\rho_g C_{pg} A_g} (t_l - t_g) \tag{1}$$

$$\frac{\partial t_l}{\partial \theta} - V_l \frac{\partial t_l}{\partial z} = \frac{UP}{\rho_l C_{pl} A_l} (t_g - t_l) - \frac{\lambda\, k_H\, P}{\rho_l C_{pl} A_l} (H'_{gi} - H'_g) \tag{2}$$

Solution of Equations (1) and (2) is complicated by the presence of the dynamic humidities H'_{gi} and H'_g. The following assumptions are made to simplify the analaysis and still include the dynamics of mass transfer:

1. The bulk gas phase humidity remains constant upon inlet gas temperature forcing, that is, $H'_g = 0$.
2. $H'_{gi} = \alpha t_i$

These two assumptions are substituted into Equation (2). Next, several time constants τ are defined and are included in the resulting gas and liquid phase equations. The following equations are obtained after taking the Laplace transformation with respect to time:

$$\frac{d\overline{t}_g}{dz} + \frac{s + (1/\tau_g)}{V_g} \overline{t}_g = \frac{1}{V_g \tau_g} \overline{t}_l \tag{3}$$

$$\frac{d\overline{t}_l}{dz} - \frac{s + (1/\tau_l) + (1/\tau_{mg})}{V_l} \overline{t}_l = \frac{(1/\tau_l) - (1/\tau_{ml})}{V_l} \overline{t}_g \tag{4}$$

Four transfer functions which relate inlet-outlet gas temperature changes are obtained by solving Equations (3) and (4) subject to various restrictions.

Model I is obtained by solving Equations (3) and (4) subject to the following split boundary conditions:

$$\bar{t}_g = \bar{t}_{g0} \quad \text{at} \quad z = 0 \tag{5}$$

$$\bar{t}_l = 0 \quad \text{at} \quad z = L \tag{6}$$

The resulting transfer function is

$$\frac{\bar{t}_{gL}}{\bar{t}_{g0}}(s) = \frac{1}{B(s)} e^{-C(s)L} \left\{ B(s) \cos [B(s)L] + \left[\frac{D \sin(B(s)L)}{B(s)\cos[B(s)L] + E(s)\sin[B(s)L]} - E(s) \right] \sin[B(s)L] \right\} \quad \text{(Model I)}$$

The coefficients $B(s)$, $C(s)$, D, and $E(s)$ in model I contain the mass transfer time constants τ_{mg} and/or τ_{ml}. Both time constants contain the linearizing constant α in the denominator. Model II, which neglects mass transfer, is obtained by letting α equal zero in model I. The mass transfer time constants approach infinity when α equals zero. Thus

$$\text{Model I}\big|_{\tau_{mg}=\tau_{ml}=\infty} = \text{Model II} \qquad \text{(Model II)}$$

Two additional models which neglect mass transfer heat effects may be derived by assuming that the liquid phase temperature change t_l is negligible compared with that of the gas phase. Model III considers the effect of gas phase temperature changes on τ_g through the linearizing expression:

$$\frac{1}{\tau_g} = \frac{1}{\tau_{gss}} \left(1 + \beta \frac{t_g}{T_{gss0}} \right) \tag{7}$$

To obtain model III, Equation (7) is first substituted into the gas phase energy balance (not included herein). The following transfer function results after lengthy manipulation:

$$\frac{\bar{t}_{gL}}{\bar{t}_{g0}}(s) = \exp \left\{ \frac{M}{K} [1 - \exp(KL)] - [F(s) + N(s)]L \right\} \quad \text{(Model III)}$$

Model IV is readily derived from Equations (3) and (4) by assuming that the mass transfer heat effect and the liquid phase temperature change t_l are negligible. Note that these assumptions reduce Equations (3) and (4) to the single equation

$$\frac{d\bar{t}_g}{dz} + \frac{s + (1/\tau_g)}{V_g} \bar{t}_g = 0 \tag{8}$$

The following transfer function is obtained:

$$\frac{\bar{t}_{gL}}{\bar{t}_{g0}}(s) = \exp - \left(\frac{s + (1/\tau_g)}{V_g} \right) L \quad \text{(Model IV)}$$

Table 1 shows the heat effects which are included in models I to IV. A detailed derivation of all models is given by Stewart (*12*).

EXPERIMENTAL

Equipment

A diagram of the experimental system is shown in Figure 1. Two 1-in. I.D. wetted-wall test sections 1½ and 3 ft. in length and of Plexiglas construction were used in the study. A detailed cross section of the 1½-ft. test section is shown in an earlier publication (*3*). Copper-constantan thermocouples at the gas stream inlet and outlet were of 30-gauge wire and sheathed in 0.072-in. O.D. hypodermic tubing. The time constant of each thermocouple was less than 0.04 sec. The shielded thermocouple leads were connected through a constant-temperature bath and a dual-channel, high-gain preamplifier to a dual-channel strip chart recorder from which the pulse traces were obtained.

The pulser consisted of a coiled 1-ft. length of 26-gauge nichrome wire and was located at the bottom of the 4-ft. calming section. Two feet upstream of the pulser the air line was divided by a tee into two parallel copper tubing air lines. Each line contained a coil and a stopcock. The stopcock handles joined in such a way that when one stopcock was exactly closed the other was exactly open. A single Dewar flask which served as an ice bath was placed around one of the coils. Heating tape was placed immediately downstream of the other coil. The two parallel air lines were joined at the upstream end with another tee.

Procedure

The air and water streams were first adjusted to the desired flow rates and temperatures. The average Reynolds number for the air phase was 4,540, while that for the liquid film (*1, 6*) was 1,145, both calculated relative to the wetted wall. The inlet air temperature was 99°F. at a humidity of 0.005 lb. H_2O vapor/lb. dry air. The inlet water temperature was 54°F., thus ensuring nonadiabatic operation.

Steady state was attained with the air flowing through the heated line. The inlet air temperature was pulsed by depressing a button on the timer which energized the nichrome pulser for ¼ sec. Just before the desired pulse amplitude was reached, the handle connecting the two stopcocks was quickly moved to change the air flow from the heated line to the cooled line. When the input pulse had closed to within ½ deg. of the steady state value, the handle was gradually moved back to its original position thus restoring the air flow to the heated line and reestablishing steady state conditions. This switching procedure significantly decreased the pulse widths obtained by using the heated air line alone. Previous investigators (*8, 10, 11*) have shown that as pulse width is reduced, reliable results may be obtained over a greater frequency range.

RESULTS AND DISCUSSION

Figure 2 shows the experimental and theoretical results for both the 1½- and 3-ft. test sections. In general, agreement between experimental and theoretical results is good.

Theoretical Results

Models I and II yielded identical amplitude ratio and phase shift results through four significant figures. Although the amplitude ratios for models I and II appear to be independent of frequency, a slight frequency dependence occurred in the third decimal place. This nonoscillatory, frequency-dependent, amplitude ratio accompanied by unlimited phase shift has been reported (*5*) for distributed-parameter, double-pipe heat exchanger dynamics where outlet tube-side temperature was forced by inlet tube-side temperature. The agreement between

TABLE 1. SUMMARY OF THERMAL EFFECTS CONSIDERED IN MODELS I TO IV

Consideration	Model I	Model II	Model III	Model IV
Gas phase temperature Change from steady state	Yes	Yes	Yes	Yes
Liquid phase temperature change from steady state	Yes	Yes	No	No
Mass transfer latent heat effect	Yes	No	No	No
Effect of temperature changes from steady state on gas phase time constant (τ_g)	No	No	Yes	No

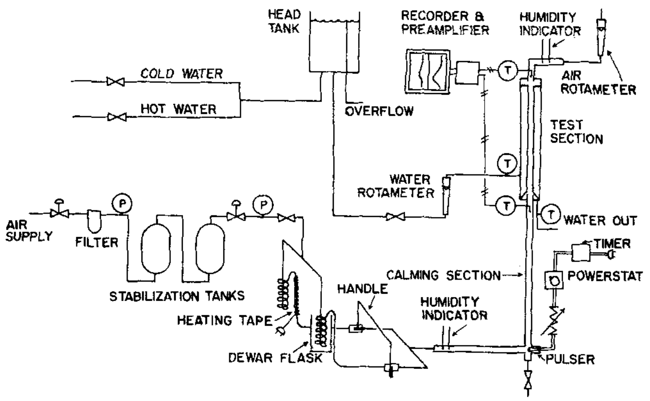

Fig. 1. Diagram of experimental apparatus.

models I and II implies that the mass transfer latent heat effect does not appreciably affect the thermal dynamics of nonadiabatic air-water contact operations of the type investigated. This conclusion was reported (2, 3) for adiabatic operation.

Figure 2 shows only a slight difference between the amplitude ratios of models II and IV. Model II differs from model IV only in that model II includes the dynamic temperature change of the liquid phase. This indicates that the change in liquid temperature from steady state is quite small compared with the change in gas temperature. This is reasonable considering that both the heat capacity and mass flow rate of the water are substantially greater than those of the air.

The slight liquid temperature change helps to explain the insignificance of the mass transfer latent heat effect. The humidity at the gas-liquid interface does not change significantly from its steady state value with only a slight change in liquid temperature. Neither does the bulk humidity of the gas phase change from its steady state value because air temperature is the forcing variable. The dynamic driving force for mass transfer is therefore negligible and mass transfer does not appreciably affect the thermal dynamics of the system.

The effect of air temperature changes upon the gas phase time constant τ_g is shown by comparison of models III and IV. The air temperature increase, upon system forcing, decreases the gas density and increases the gas phase velocity and film coefficient and the heat transfer rate. The gas phase time constant is therefore decreased. This decreased time constant results in a smaller change in outlet air temperature for a given change in inlet air temperature and hence a lower amplitude ratio for model III compared with model IV.

Theoretical results for the simplest model, model IV, agree closely with the more complex models I and II. This result is significant but not totally unexpected for the air-water system. Equations (3) and (4), from which models I and II are obtained, reduce to Equation (8), from which model IV is obtained, with the assumption of a negligible mass transfer heat effect and liquid temperature change. Small changes in liquid temperature and mass transfer rate for the air-water system are reasonable due to the high heat capacity and mass flow rate of the liquid phase relative to the gas phase.

All models gave the same phase shift results. The phase shift predicted by model IV is clearly due to pure transport delay. Therefore, the phase shift for the air-water system is pure transport delay.

Experimental Results

Frequency response results were obtained by numerical Fourier transformation of the experimental input and output pulses (4) by using Filon's modification (7) of Simpson's rule to evaluate the integrals. The amplitude ratio for the wetted-wall test section was obtained by dividing the amplitude ratio for the total column (wetted-wall test section plus the inlet and outlet sections) by the amplitude ratio for the inlet-outlet section. The phase shift for the wetted-wall test section was obtained by subtracting the phase shift for the inlet-outlet section from that of the total column.

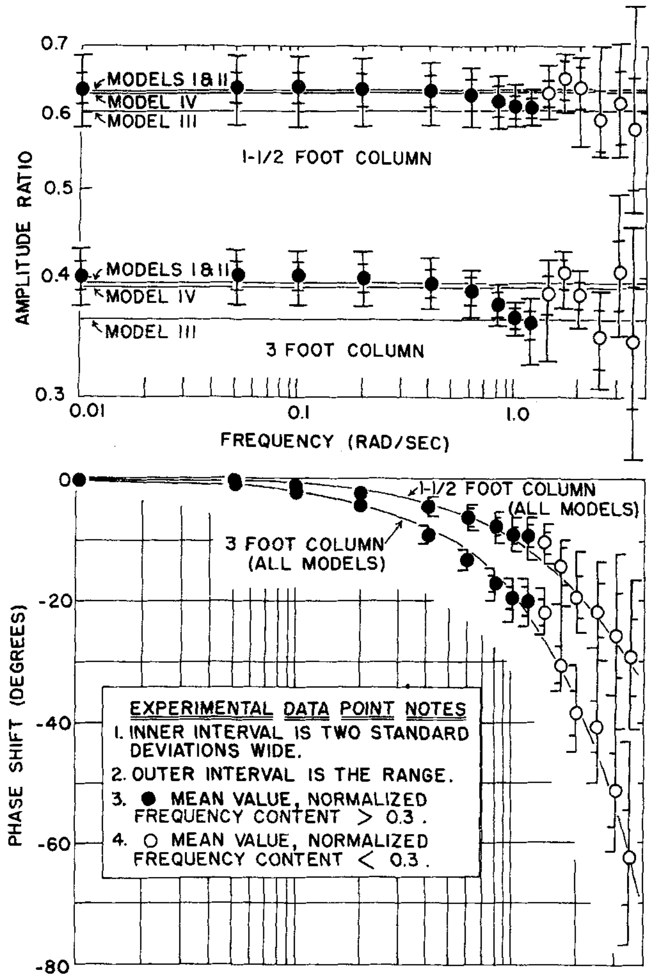

Fig. 2. Comparison of experimental and theoretical results.

Six pulse tests were made for the 1½- and 3-ft. total columns and the inlet-outlet section. Since six experimental values of amplitude ratio and phase shift were obtained for each total column and the inlet-outlet section, it was possible to obtain thirty-six values of amplitude ratio and phase shift for each column at each frequency. Each dot and circle in Figure 2 represents the mean value of these thirty-six experimental points. The dispersion about the mean value is indicated by the inner interval shown on nearly all data points which is two standard deviations wide. The outer interval shows the range of the thirty-six-point sample. The similarity between the data scattering patterns for the 1½- and 3-ft. test sections is largely due to the fact that the same set of six inlet-outlet pulse tests was used to obtain frequency response data for both test sections.

The standard deviation interval includes amplitude ratio values within about ± 4.0% of the mean value for frequencies at which the normalized frequency content is above 0.3. This agreement, coupled with the fact that input pulse amplitudes were between only 3.0 and 5.0°F., suggests that nonlinearities were not excited.

Figure 2 shows an increase, with frequency, of the standard deviation and range intervals for normalized frequency contents below 0.3. Data scatter within the 0.0 to 0.3 normalized frequency content range has been reported by previous investigators (*4, 10, 11*).

Uniqueness of Experimental Results

The importance of maintaining all system parameters other than the forcing variable at their steady state values has been cited (*9*). Pulse testing results will not be independent of input pulse characteristics unless this is done. In this study the air velocity did not remain at its steady state value for the duration of the input temperature pulse. First, upon pulsing, the heat generated at the nichrome pulser momentarily increased the air velocity by about 3 to 4%. Introduction of cold air when the desired pulse amplitude was attained resulted in a sudden velocity decrease from the steady state value of approximately 2%. Subsequent return of air flow to the heated line returned the air velocity to its steady state value.

The air velocity change which occurred during pulsing was simulated by manual operation of a valve in the air line in order to obtain an estimate of this velocity forcing effect. Results indicated that this superimposed velocity forcing effect increased the experimental amplitude ratio by 2%, at most. Since this effect was not accurately determined, the experimental points of Figure 2 were not adjusted to eliminate this slight velocity forcing effect.

CONCLUSIONS

For the conditions studied, the following conclusions may be drawn for nonadiabatic humidification in a wetted-wall column.

1. Model IV, the simplest model, as well as models I to III adequately describe the frequency response characteristics of nonadiabatic humidification in systems of the type studied.
2. Mass transfer and the change in liquid phase temperature do not appreciably affect the thermal dynamic behavior of nonadiabatic humidification under the conditions of gas phase temperature forcing studied here.
3. Only models I and II yield frequency-dependent amplitude ratios. This may be more important for significantly longer columns and/or for flow conditions at which the overall heat transfer coefficient is appreciably higher than that of the present study.
4. The system phase shift is due to pure transport delay.
5. Amplitude ratio and phase shift data scatter begins at a normalized frequency content of about 0.3 and becomes severe below 0.2.
6. Additional studies should be made to determine if the averaging procedure which yielded apparently accurate mean values of amplitude ratio and phase shift at normalized frequency contents as low as 0.08 is fortuitous or is a generally applicable technique.

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NOTATION

A = cross-sectional area, sq.ft.
B = $(D - E^2)^{1/2}$
C = $(F - G)/2$
C_p = heat capacity, B.t.u./(lb.) (°F.)
D = $\{1/(\tau_g \tau_l V_g V_l)\} + \{1/(\tau_g \tau_{ml} V_g V_l)\}$
E = $(F + G)/2$
F = $(s + (1/\tau_g))/V_g$
G = $(s + (1/\tau_l) + (1/\tau_{mg}))/V_l$
h = film heat transfer coefficient, B.t.u./(hr.) (sq.ft.) (°F.)
H' = dynamic humidity, lb. vapor/lb. dry gas
I = $(T_{lss0} - T_{lssL} - T_{gss0} + T_{gssL})/(1 - \exp(KL))$
J = $\{(T_{lssL} - T_{gssL}) + (T_{gss0} - T_{lss0}) \exp(KL)\}/\{1 - \exp(KL)\}$
K = $\{1/(\tau_{lss} V_l)\} - \{1/(\tau_{gss} V_g)\}$
k_H = gas phase mass transfer coefficient, lb./(hr.) (sq.ft.) (ΔH)
L = length of wetted-wall test section, ft.
M = $\beta J/(\tau_{gss} V_g T_{gss0})$
N = $\beta I/(\tau_{gss} V_g T_{gss0})$
P = circumference of gas-liquid interface, ft.
s = Laplace transform variable
t = temperature change from steady state, °F.
$\bar{t}$ = Laplace transformed temperature change
T = temperature, °F.
U = overall heat transfer coefficient, B.t.u./(hr.) (sq. ft.) (°F.)
U = $h_g h_l/(h_g + h_l)$
V = velocity, ft./hr.
z = axial position, ft.

Greek Letters

α = constant relating change in interface liquid temperature to change in interface humidity, (lb. vapor/lb. dry air)/°F.
β = dimensionless constant relating change in gas phase time constant to change in gas phase temperature
θ = time, hr.
ρ = density, lb.$_m$/cu.ft.
λ = latent heat of vaporization, B.t.u./lb.
τ_g = $\rho_g C_{pg} A_g/(UP)$, hr.
τ_l = $\rho_l C_{pl} A_l/(UP)$, hr.
τ_{mg} = $h_g \tau_l/(\lambda k_H \alpha)$, hr.
τ_{ml} = $h_l \tau_l/(\lambda k_H \alpha)$, hr.

Subscripts

g = gas phase
l = liquid phase
i = gas-liquid interface
ss = steady state
m = mass transfer
0 = air inlet
L = air outlet

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